FST 10.1 Notes
Topic: Combinations
GOAL
Exhibit situations where combinations are desired. Derive a formula for the number of combinations of n things r at a time.

F Use combinations to compute the number of ways of selecting objects.
G Compute probabilities involving combinations in non-binomial problems.

Vocabulary

combination
number of combinations of n things taken r at a time, ${}_nC_r \binom{n}{r}$

Warm up
In the 26-letter English alphabet, how many different 2-letter initials are possible:

a) If the same letter can be repeated? $26 \cdot 26 = 676$

b) If the same letter cannot be repeated? $26 \cdot 25 = 650$

Permutations = is the arrangement of n things taken r at a time. - order matters

Theorem (Alternate Formula for ${}_nP_r$)

$${}_nP_r = \frac{n!}{(n-r)!}$$

MATH → PRB #2

Combination = is the number of n things taken r at a time. - order Does Not matter

${}_nC_r$ or $\binom{n}{r}$

Theorem (Formula for ${}_nC_r$)

For all integers n and r, with $0 \le r \le n$, ${}_nC_r = \frac{{}_nP_r}{r!} = \frac{n!}{(n-r)!r!}$.

$\frac{n!}{(n-r)!r!}$

Evaluate. Know the formula

${}_{10}C_6 \quad \frac{10!}{(10-6)!6!} = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{5040}{24} = 210$

$17C_3 = \binom{17}{3} \quad \frac{17!}{(17-3)!3!} = \frac{17!}{14!3!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1}$

$\frac{17 \cdot 16 \cdot 15}{3 \cdot 2 \cdot 1} = \frac{4080}{6} = 680$

Example 1 P C
Determine whether the problem involves permutations or combinations.

a) How many ways are there to choose a committee of 2 people from a group of 7 people? C $7C_2$

b) How many ways are there to choose a chairperson and a co-chairperson from a group of 7 people? P $7P_2$

c) How many ways are there to award first prize, second prize, and third prize to a group of 8 entrants in a contest? P $8P_3$

d) How many ways are there to give 3 honorable mention awards to a group of 8 entrants in a contest? C $8C_3$

Example 2 25 total
A class has 10 boys and 15 girls. A committee of 5 is to be chosen to represent the class at a group meeting, so it is decided to pick 2 boys and 3 girls. How many different committees are possible? **Find the probability that out of the whole class, 5 students would be chosen and exactly 2 of them are boys and 3 are girls.**

$10C_2 \cdot 15C_3 = 20{,}475$ committees

$$\frac{10C_2 \cdot 15C_3}{25C_5} = \frac{20{,}475}{53{,}130} = 38.5\%$$

Example 3
Bill drives to school and the route he takes has 5 stoplights. If he reaches all the lights on green, then it takes him 12 minutes to get to school. But every stoplight he reaches on red slows him down about a minute. In how many ways can he be slowed down to get to school in about 15 minutes?

15-12 = 3 extra minute

$5C_3 = 10$ ways

Example 4
To win a particular lottery game, it is necessary to match the numbers on 5 balls that are randomly picked from 50 balls numbered 1 to 50 and the number on a special black ball that is picked from 9 other balls numbered 1 to 9. What is the probability that a single lottery ticket will match the 5 balls and the special black ball?

$50C_5 \cdot 9C_1 = 19{,}068{,}840$

$$\frac{1}{19{,}068{,}840} = 5.24 \times 10^{-8}$$

$= .0000000524 = .000005\%$